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Aharonov–Casher effect for spin-1 particles in a non-commutative space

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Abstract. In this work, the Aharonov–Casher (AC) phase is calculated for spin-1 particles in a non-commutative space. The AC phase has previously been calculated from the Dirac equation in a non-commutative space using a gauge-like technique. In the spin-1 case, we use the Kemmer equation to calculate the phase in a similar manner. It is shown that the holonomy receives non-trivial kinematical corrections. By comparing the new result with the already known spin-1/2 case, one may conjecture a generalized formula for the corrections to holonomy for higher spins.

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1 Introduction

In the last few years, theories in non-commutative space have been studied extensively (for a review see [1]). Non-commutative field theories are related to M-theory compactification [2], string theory in non-trivial backgrounds [3] and the quantum Hall effect [4]. Inclusion of non-commutativity in quantum field theory can be achieved in two different ways: via the Moyal $*$-product on the space of ordinary functions, or via defining the field theory on a coordinate operator space which is intrinsically non-commutative [1, 5]. The equivalence between the two approaches has been nicely described in [6]. A simple insight on the role of non-commutativity in field theory can be obtained by studying the one particle sector, which prompted interest in the study of non-commutative quantum mechanics [7–14]. In these studies some attention was paid to the Aharonov–Bohm effect [15]. If the non-commutative effects are important at very high energies, then one could posit a decoupling theorem that produces the standard quantum field theory as an effective field theory and that does not recall the non-commutative effects. However, the experience from atomic and molecular physics strongly suggests that the decoupling is never complete and that the high energy effects appear in the effective action as topological remnants. Along these lines, the Aharonov–Bohm and Aharonov–Casher effects have already been investigated in a non-commutative space [16, 17]. In this work, we will develop a method to obtain the corrections to the topological phase of the Aharonov–Casher effect for spin-1 particles, where we know that in a commutative space the line spectrum does not depend on the relativistic nature of the dipoles. The article is organized as follows; in Sect. 2, we discuss the Aharonov–Casher effect for spin-1 particles on a commutative space. In Sect. 3, the Aharonov–Casher effect in a non-commutative space is studied and a generalized formula for holonomy is given.

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2 The Aharonov–Casher effect

In 1984 Aharonov and Casher (AC) [18] pointed out that the wave function of a neutral particle with non-zero magnetic moment μ develops a topological phase when traveling in a closed path which encircles an infinitely long filament carrying a uniform charge density. The AC phase has been measured experimentally [19]. This phenomenon is similar to the Aharonov–Bohm (AB) effect. The similarities and the differences of these two phenomena and possible classical interpretations of the AC effect have been discussed by several authors [20–22]. In [18], the topological phase of the AC effect was derived by considering a neutral particle with a non-zero magnetic dipole moment moving in an electric field produced by an infinitely long filament carrying a uniform charge density. If the particle travels over a closed path which includes the filament, a topological phase will result. This phase is given by

$$\phi_{\mathrm{AC}} = \oint (\boldsymbol{\mu} \times \mathbf{E})\,\mathrm{d}\mathbf{r}\,, \tag{1}$$

where $\boldsymbol{\mu} = \mu\boldsymbol{\sigma}$ is the magnetic dipole moment and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$, where σ_i $(i = 1, 2, 3)$ are the 2×2 Pauli matrices. It is possible to arrange that the particle moves in the x–y plane and travels over a closed path which includes an infinite filament along the z-axis. The electric field in the point $\mathbf{r} = x\hat{\imath} + y\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the direction of the positive x and y axes, is given by

$$\mathbf{E} = \frac{\lambda}{2\pi(x^2+y^2)}(x\hat{\imath}+y\hat{\jmath})\,, \tag{2}$$

where λ is the linear charge density of the filament, and the phase is given by

$$\phi_{\mathrm{AC}} = \mu\sigma_3 \oint (\hat{k}\times\mathbf{E})\cdot \mathrm{d}\mathbf{r} = \mu\sigma_3\lambda\,, \tag{3}$$

where $\hat{k}$ is a unit vector along the z-axis. This phase is purely quantum mechanical and has no classical interpretation. The appearance of σ_3 in the phase represents the spin degrees of freedom. We see that different components acquire phases with different signs. This is also one of the points that distinguishes the AC effect from the AB effect [23]. In this part, we briefly explain a method for obtaining (3). The equation of motion for a neutral spin-1/2 particle with a non-zero magnetic dipole moment moving in a static electric field $\mathbf{E}$ is given by

$$\left(\mathrm{i}\gamma_\mu\partial^\mu + \frac{1}{2}\mu\sigma_{\alpha\beta}F^{\alpha\beta} - m\right)\psi = 0, \tag{4}$$

or it can be written as

$$(\mathrm{i}\gamma_\mu\partial^\mu - \mathrm{i}\mu\gamma\cdot\mathbf{E}\gamma_0 - m)\psi = 0\,, \tag{5}$$

where $\gamma = (\gamma^1,\gamma^2,\gamma^3)$ and the γ-matrices are defined by

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}\,. \tag{6}$$

We define

$$\psi = \mathrm{e}^{\mathbf{a}f}\psi_0\,, \tag{7}$$

where $\mathbf{a}$ is the matrix to be determined below, f is a time independent scalar phase, and ψ_0 is a solution of the Dirac equation

$$(\mathrm{i}\gamma_\mu\partial^\mu - m)\psi_0 = 0\,. \tag{8}$$

Writing ψ_0 in terms of ψ and multiplying (8) by $\mathrm{e}^{\mathbf{a}f}$ from the left, we obtain

$$\mathrm{e}^{\mathbf{a}f}\,(\mathrm{i}\gamma^\mu\partial_\mu - m)\,\mathrm{e}^{-\mathbf{a}f}\psi = 0 \tag{9}$$

$$\left(\mathrm{i}\mathrm{e}^{\mathbf{a}f}\gamma^\mu\mathrm{e}^{-\mathbf{a}f}\partial_\mu - \mathrm{i}\mathrm{e}^{\mathbf{a}f}\gamma^i\mathrm{e}^{-\mathbf{a}f}\mathbf{a}\partial_i f - m\right)\psi = 0\,. \tag{10}$$

Comparing (10) with (5), we find that $\mathbf{a}$ and f must satisfy

$$\mu\gamma\cdot\mathbf{E}\gamma_0 = (\gamma\cdot\nabla f)\mathbf{a}\,, \quad \mathbf{a}\gamma_\mu = \gamma_\mu\mathbf{a}\,. \tag{11}$$

The matrix $\mathbf{a}$ can be expressed by some linear combination of the complete set of 4×4 matrices $1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5$ and $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu,\gamma_\nu]$. The second member of (11) cannot be satisfied if all γ_1,γ_2 and γ_3 are present in (10). However, it is possible to satisfy it if the problem in question can be reduced to the planar one. This indicates that the AC topological phase can arise only in two spatial dimensions. Therefore, let us consider the particle moving in the x–y plane in which case only the matrices γ_1 and γ_2 are present in (11), and moreover, $\partial_3\psi$ and E_3 vanish. The choice $-\mathrm{i}\sigma_{12}\gamma_0$ represents a consistent Ansatz. From the first equation in (11), we get

$$\nabla f = \mu(\hat{k}\times\mathbf{E}), \tag{12}$$

and the phase is given by

$$\begin{aligned}\phi^{(0)} &= \sigma_{12}\gamma_0 \oint \nabla f\cdot\mathrm{d}\mathbf{r} \\ &= \mu\sigma_{12}\gamma_0 \oint (\hat{k}\times\mathbf{E})\cdot\mathrm{d}\mathbf{r} \\ &= \mu\begin{pmatrix}\sigma_3 & 0 \\ 0 & -\sigma_3\end{pmatrix}\oint(\hat{k}\times\mathbf{E})\cdot\mathrm{d}\mathbf{r}\,.\end{aligned} \tag{13}$$

One may extend this method to spin 1 using the first order Kemmer equation (for a more complete explanation and derivation see [24]). The Kemmer equation is defined by

$$(\mathrm{i}\beta^\mu\partial_\mu - m)\psi = 0\,, \tag{14}$$

where the β-matrices are generalizations of the Dirac gamma matrices. These satisfy an algebra ring, which for spin 1 is

$$\beta^\mu\beta^\nu\beta^\rho + \beta^\rho\beta^\nu\beta^\mu = \eta^{\mu\nu}\beta^\rho + \eta^{\nu\rho}\beta^\mu\,. \tag{15}$$

These Kemmer β-matrices are reducible; that is, the 16×16 representation decomposes into three separate representations: a one dimensional trivial representation; a five dimensional spin zero representation; and the $10-d$ spin-1 representation [24–26]. This algebra is odd; that is, it cannot reduce the matrix operator to the identity, unlike the Dirac algebra. In this paper we choose the $10-d$ spin-1 representation which is given by the following 10×10 matrices:

$$\beta^0 = \begin{pmatrix} \widehat{O} & \widehat{O} & I & o^\dagger \\ \widehat{O} & \widehat{O} & \widehat{O} & o^\dagger \\ I & \widehat{O} & \widehat{O} & o^\dagger \\ o & o & o & 0 \end{pmatrix}$$
$$\beta^j = \begin{pmatrix} \widehat{O} & \widehat{O} & \widehat{O} & -\mathrm{i}K^{j\dagger} \\ \widehat{O} & \widehat{O} & S^j & o^\dagger \\ \widehat{O} & -S^j & \widehat{O} & o^\dagger \\ -\mathrm{i}K^j & o & o & 0 \end{pmatrix}, \tag{16}$$

where the elements are

$$\widehat{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{17}$$

$$S^1 = \mathrm{i}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^2 = \mathrm{i}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},$$

$$S^2 = \mathrm{i}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{18}$$

$$o = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix},$$
$$K^1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad K^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \quad K^3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}. \tag{19}$$

Just as in Dirac theory, the Lorentz invariance of the Kemmer theory entails a transformation of the spinor so that the matrix representation remain the same. The Lorentz generator for these transformations, $S_{\mu\nu}$, is proportional to the antisymmetric product of two matrices of the ring,

$$S_{\mu\nu} = b(\beta_\mu\beta_\nu - \beta_\nu\beta_\mu). \tag{20}$$

These generators satisfy the well known commutation relation and define the spin operators. The coefficient b is linked to the coefficient of the commutation relations and is set below according to our convenience. The equation of motion for a neutral spin-1 particle with an anomalous magnetic moment in Kemmer theory is

$$\left(\mathrm{i}\beta_\mu\partial^\mu + \frac{1}{2}\mu S_{\alpha\beta}F^{\alpha\beta} - m\right)\psi = 0. \tag{21}$$

The interaction term emerges from the derivation of a second order Kemmer equation following the method of Umezawa [26]. The operator component of the phase in the spin-1 AC phase solution is a spin-1 pseudo-vector operator defined by

$$\xi_\mu = \frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\lambda\rho}\beta^\nu\beta^\lambda\beta^\rho. \tag{22}$$

The path dependent phase proportional to ξ_3 is introduced in the free Kemmer equation of motion [17] thus

$$(\mathrm{i}\beta^\mu\partial_\mu - m)\,\mathrm{e}^{\mathrm{i}\xi_3\int^r \mathbf{A}\cdot\mathrm{d}\mathbf{r}}\psi = 0, \tag{23}$$

with the intention to transform this into the equation of motion (21) with the anomalous magnetic moment term. Multiplying (23) by $\mathrm{e}^{(-\mathrm{i}\xi_3\int^r \mathbf{A}\mathrm{d}\mathbf{r})}$ from the left and comparing with (21), we obtain

$$\exp\left[-\mathrm{i}\xi_3\int^r \mathbf{A}\cdot\mathrm{d}\mathbf{r}\right]\beta^\nu\exp\left[\mathrm{i}\xi_3\int^r \mathbf{A}\cdot\mathrm{d}\mathbf{r}\right] = \beta^\nu, \tag{24}$$

$$-\beta^\nu\xi_3 A_\nu\psi = \frac{1}{2}\mu S_{\alpha\beta}F^{\alpha\beta}\psi = \mu S_{0i}F^{0i}\psi. \tag{25}$$

By using the Baker–Hausdorff formula in the first condition, it is easy to see that for $\nu \neq 3$ the commutators are zero. However for $\nu = 3$ the commutators do not vanish and so the first condition restricts the dynamics to $2+1$ dimensions, just as the spin-1/2 case.

By a direct calculation in the second line and using the definition of the ξ_3-, β^ν- and $S_{\mu\nu}$-matrices one has

$$A_1 = -2\mu E_2, \quad A_2 = 2\mu E_1. \tag{26}$$

Finally the AC phase for a closed path is given by

$$\phi_{\mathrm{AC}} = \xi_3\oint \mathbf{A}\cdot\mathrm{d}\mathbf{r} = 2\mu\xi_3\int_S(\nabla\cdot E)\,\mathrm{d}S = 2\mu\xi_3\lambda. \tag{27}$$

3 The spin-1 AC effect on a non-commutative space

The non-commutative Moyal spaces can be realized as spaces where the coordinate operator $\hat{x}^\mu$ satisfies the commutation relations

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}, \tag{28}$$

where $\theta^{\mu\nu}$ is an antisymmetric tensor and is of space dimension $(\text{length})^2$. We note that space-time non-commutativity, $\theta^{0i} \neq 0$, may lead to some problems with unitarity and causality. Such problems do not occur for quantum mechanics on a non-commutative space with a usual commutative time coordinate. The non-commutative models specified by (14) can be realized in terms of a $*$-product: the commutative algebra of functions with the usual product $f(x)g(x)$ is replaced by the $*$-product Moyal algebra:

$$(f*g)(x) = \exp\left[\frac{\mathrm{i}}{2}\theta_{\mu\nu}\partial_{x_\mu}\partial_{y_\nu}\right]f(x)g(y)|_{x=y}. \tag{29}$$

As for the phase space, as inferred from string theory, we choose

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\theta_{ij}, \quad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\hbar\delta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0. \tag{30}$$

The non-commutative quantum mechanics can be defined by [7–14]

$$H(p,x)*\psi(x) = E\psi(x). \tag{31}$$

The equation of motion for a neutral spin-1 particle with a non-zero magnetic dipole moment moving in a static electric field $\mathbf{E}$ is given by

$$\left(\mathrm{i}\beta_\mu\partial^\mu + \frac{1}{2}\mu S_{\alpha\beta}F^{\alpha\beta} - m\right)*\psi = 0. \tag{32}$$

As in [17] we define

$$\psi = \mathrm{e}^{\mathrm{i}\xi_3 f}\psi_0, \tag{33}$$

where ξ_3 is the matrix already defined, f is a time independent scalar phase, and ψ_0 is a solution of the free Kemmer equation

$$(\mathrm{i}\beta_\mu\partial^\mu - m)\psi_0 = 0. \tag{34}$$

By using the Baker–Campłe–Hausdorff formula and $[\beta_\mu, \xi_3] = 0$, (32) can be written as

$$-\beta^\mu \partial_\mu(\xi_3 f)\mathrm{e}^{\mathrm{i}\xi_3 f}\psi_0 + \frac{\mu}{2}S_{\alpha\beta}F^{\alpha\beta} * \left(\mathrm{e}^{\mathrm{i}\xi_3 f}\psi_0\right) = 0\,. \tag{35}$$

After expanding the second term in (35) up to the first order of the non-commutativity parameter $\theta_{ij} = \theta\epsilon_{ij}$ and defining k_j as

$$\partial_j\psi_0 = (\mathrm{i}k_j)\psi_0 \tag{36}$$

the final result up to first order in θ is given by

$$\left[-\beta^\mu\partial_\mu(\xi_3 f) + \mu\left(S_{0l}F^{0l} + \frac{\mathrm{i}}{2}\theta_{ij}\partial_i\left(S_{0l}F^{0l}\right)\partial_j(\mathrm{i}\xi_3 f) + \frac{\mathrm{i}}{2}\theta_{ij}\partial_i\left(S_{0l}F^{0l}\right)(\mathrm{i}k^j)\right)\right]\exp[i\xi_3 f]\psi_0 = 0\,. \tag{37}$$

It should be noted that the expansion of F^{0l} or $\mathbf{E}$ up to first order in θ leads to an additive correction to the commutative holonomy and does not cause a new non-topological behavior. A similar situation occurs in the non-commutative Aharonov–Bohm effect. By expanding f up to first order in θ,

$$f = f^{(0)} + \theta f^{(1)} + \dots\,, \tag{38}$$

we obtain the following equations:

$$\left[-\beta^\mu\partial_\mu\left(\xi_3 f^{(0)}\right) + \mu\left(S_{0l}F^{0l}\right)\right]\psi_0 = 0 \tag{39}$$

$$\left[-\beta^\mu\partial_\mu\left(\xi_3 f^{(1)}\right) + \frac{\mathrm{i}\mu}{2}\varepsilon_{ij}\partial_i\left(S_{0l}F^{0l}\right)\partial_j\left(\mathrm{i}\xi_3 f^{(0)}\right) + \frac{\mathrm{i}\mu}{2}\varepsilon_{ij}\partial_i\left(S_{0l}F^{0l}\right)(\mathrm{i}k^j)\right]\psi_0 = 0\,, \tag{40}$$

by choosing $b = 2$ in (20), and after a straightforward calculation, we get

$$\nabla f^{(0)} = 2\mu(\hat{k}\times\mathbf{E})\,, \tag{41}$$

which is equivalent to (26), and the phase is given by

$$\phi^{(0)} = \xi_3\oint\nabla f^{(0)}\,\mathrm{d}\mathbf{r} = 2\mu\xi_3\oint(\hat{k}\times\mathbf{E})\,\mathrm{d}\mathbf{r}\,. \tag{42}$$

Substituting (42) in (40) and then using the wave functions which are given in [24], a long but straightforward calculation (the Mathematica package is used) yields the following correction to $\phi^{(0)}$ for a neutral particle with non-zero magnetic dipole moment μ and with spin 1 ($m_s = 1, 0, -1$):

$$\begin{aligned}\triangle\phi_\theta &= \theta\xi_3\oint\nabla f^{(1)}\,\mathrm{d}\mathbf{r}\\ &= \frac{\theta}{2}\xi_3\varepsilon^{ij}\bigg(\mu\oint k_j(\partial_i E_2\,\mathrm{d}x_1 - \partial_i E_1\,\mathrm{d}x_2)\\ &\quad - 2m_s\oint[(\mu\partial_i E_2)\mu(\hat{k}\times\mathbf{E})_j\,\mathrm{d}x_1\\ &\quad - (\mu\partial_i E_1)\mu(\hat{k}\times\mathbf{E})_j\,\mathrm{d}x_2]\bigg)\,,\end{aligned} \tag{43}$$

where $m_s = 1, 0, -1$. The first term is a velocity dependent correction and does not have the topological properties of the commutative AC effect and could modify the phase shift. The second term is a correction to the vortex and does not contribute to the line spectrum. Using $m_s = 1/2, -1/2$ the integral in (43) can be mapped to the corrections which have already been obtained for the spin-1/2 Aharonov–Casher effect in [17], (32). One may conjecture that (43) is also valid for higher spins. It is interesting to extend these results to higher order terms; however, it seems that obtaining an exact result similar to the commutative case is not possible by these methods. For some other interesting relevant papers see [27–30].

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